

Magnetic Field Induced Charging Effects in Josephson Junction Arrays

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A magnetic field induced electric polarization and the corresponding change of an effective junction capacitance are considered within a 3D model of disordered Josephson junction arrays. At some threshold field (near the Josephson network critical field), the effective junction charge and the related capacitance are shown to reach a maximum and to change a sign, respectively. A possibility to observe the predicted effects in artificially prepared arrays of superconducting grains is discussed.

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Recently, a variety of different field-induced phenomena in high- T_c superconductors (HTS), related to their extrinsic or intrinsic granularity, have been reported (see, e.g., [1–6] and further references therein). In particular, an appearance of an electric-field induced non-zero magnetization signal in granular superconductors (due to the Dzyaloshinski-Moria type interaction between an applied electric field and an effective magnetic field of circulating Josephson currents) was predicted [6]. At the same time, an artificially prepared islands of superconducting grains, well-described by the various models of Josephson junction arrays (JJA), proved useful in studying (both theoretically and experimentally) the charging effects in these systems, ranging from Coulomb blockade of Cooper pair tunneling and Bloch oscillations [7,8] to propagation of quantum ballistic vortices [9].

The present paper addresses yet another related phenomenon which is actually dual to the above-mentioned analog of magnetoelectric effect (described in Ref. [6]). Specifically, we discuss a possible appearance of a non-zero electric polarization and the related change of the charge balance in the system of weakly-coupled superconducting junctions (modelled by the random 3D JJAs) under the influence of an applied magnetic field.

To achieve our goal, we apply the so-called model of disordered 3D JJAs based on the well-known tunneling Hamiltonian (see, e.g., [10])

$$\mathcal{H} = \sum_{ij}^N J_{ij} [1 - \cos \phi_{ij}(\vec{H})], \quad (1)$$

where

$$\phi_{ij}(\vec{H}) = \phi_{ij}(0) - A_{ij}(\vec{H}), \quad (2)$$

with

$$\phi_{ij}(0) = \phi_i - \phi_j, \quad (3)$$

and

$$A_{ij}(\vec{H}) = \frac{\pi}{\Phi_0} (\vec{H} \times \vec{R}_{ij}) \vec{r}_{ij}, \quad (4)$$

where

$$\vec{r}_{ij} = \vec{r}_i - \vec{r}_j, \quad \vec{R}_{ij} = (\vec{r}_i + \vec{r}_j)/2, \quad (5)$$

which describes an interaction between N superconducting grains (with phases ϕ_i), arranged in a random three-dimensional (3D) lattice with coordinates $\vec{r}_i = (x_i, y_i, z_i)$. The grains are separated by insulating boundaries producing Josephson coupling with energy $J_{ij} = J$. The system is under the influence of a frustrating applied magnetic field \vec{H} .

Since the field-induced effects considered in the present paper are expected to manifest themselves in high enough applied magnetic fields (with a nearly homogeneous distribution of magnetic flux along the junctions) and as long as the Josephson penetration length λ_J exceeds the characteristic size of the Josephson network d (which is related to the projected junction area S , where the field penetration actually occurs, as follows $S = \pi d^2$), the Josephson current-induced "self-field" effects (which are important for a large-size junctions and/or small applied magnetic fields) may be safely neglected [11]. Besides, it is known [12] that in discrete JJAs pinning (by a single junction) actually concurs with the "self-field" effects. Specifically, it was found [12] that the ratio d/λ_J is related to the dimensionless pinning strength parameter β as $d/\lambda_J = \sqrt{\beta}$ suggesting that a weak pinning regime (with $\beta \ll 1$) simultaneously implies a smallness of the "self-field" effect and vice versa. And since artificially prepared Josephson networks allow for more flexibility in varying the experimentally-controlled parameters, it is always possible to keep the both above effects down by appropriately tuning the ratio d/λ_J . Typically [7–9], in this kind of experiments $S = 0.01 - 0.1 \mu\text{m}^2$ and $d \ll \lambda_J$.

In what follows, we are interested in the magnetic field induced behavior of the electric polarization in a 3D JJA at zero temperature. Recall that a conventional (zero-field) pair polarization operator within the model under discussion reads [13]

$$\vec{p} = \sum_{i=1}^N q_i \vec{r}_i, \quad (6)$$

where $q_i = -2en_i$ with n_i the pair number operator, and r_i is the coordinate of the center of the grain.

In view of Eqs.(1)-(6), and taking into account a usual "phase-number" commutation relation, $[\phi_i, n_j] = i\delta_{ij}$, the evolution of the pair polarization operator is determined via the equation of motion

$$\frac{d\vec{p}}{dt} = \frac{1}{i\hbar} [\vec{p}, \mathcal{H}] = \frac{2e}{\hbar} \sum_{ij}^N J \sin \phi_{ij}(\vec{H}) \vec{r}_{ij} \quad (7)$$

Resolving the above equation, we arrive at the following net value of the magnetic-field induced polarization (per grain)

$$\begin{aligned} \vec{P}(\vec{H}) &\equiv \frac{1}{N} \overline{\langle \vec{p}(t) \rangle} \\ &= \frac{2eJ}{\hbar\tau N} \int_0^\tau dt \int_0^t dt' \sum_{ij}^N \langle \sin \phi_{ij}(\vec{H}) \vec{r}_{ij} \rangle, \end{aligned} \quad (8)$$

where $\langle \dots \rangle$ denotes a configurational averaging over the grain positions, while the bar means a temporal averaging (with a characteristic time τ). To consider a field-induced polarization only, we assume that in a zero magnetic field, $\vec{P} \equiv 0$, implying $\phi_{ij}(0) \equiv 0$.

To obtain an explicit form of the field dependence of polarization, let us consider a site-type positional disorder allowing for weak displacements of the grain sites from their positions of the original 3D lattice, i.e., within a radius d (which is of the order of the Josephson lattice parameter and the characteristic junction size, see above) the new position is chosen randomly according to the normalized distribution function $f(\vec{r}, \vec{R})$. It can be shown that a particular choice of the distribution function will not change the main qualitative results of the present paper. So, assuming, for simplicity, the following distribution law

$$f(\vec{r}, \vec{R}) = f_r(\vec{r}) f_R(\vec{R}), \quad (9)$$

with

$$f_r(\vec{r}) = \frac{1}{(2\pi d^2)^{3/2}} e^{-(x^2+y^2+z^2)/2d^2}, \quad (10)$$

and

$$f_R(\vec{R}) = \delta(X-d)\delta(Y-d)\delta(Z-d), \quad (11)$$

we observe that the magnetic field $\vec{H} = (0, 0, H_z)$ (applied along the z -axis) will induce a non-vanishing longitudinal (along x -axis) electric polarization

$$P_x(H_z) = P_0 G(H_z/H_0), \quad (12)$$

with

$$G(z) = ze^{-z^2} \quad (13)$$

Here $P_0 = e\tau J/\hbar$, $H_0 = \Phi_0/S$ with $S = \pi d^2$ being an average projected area of a single junction, and $z = H_z/H_0$.

For small applied fields ($z \ll 1$), the induced polarization $P_x(H_z) \approx \alpha_0 H_z$, where $\alpha_0 = e\tau d J/\hbar H_0$ is the so-called linear magnetoelectric coefficient [14]. However, as we mentioned in the very beginning, to correctly describe any induced effects in very small external fields,

both Josephson junction pinning and "self-field" effects have to be taken into account [12].

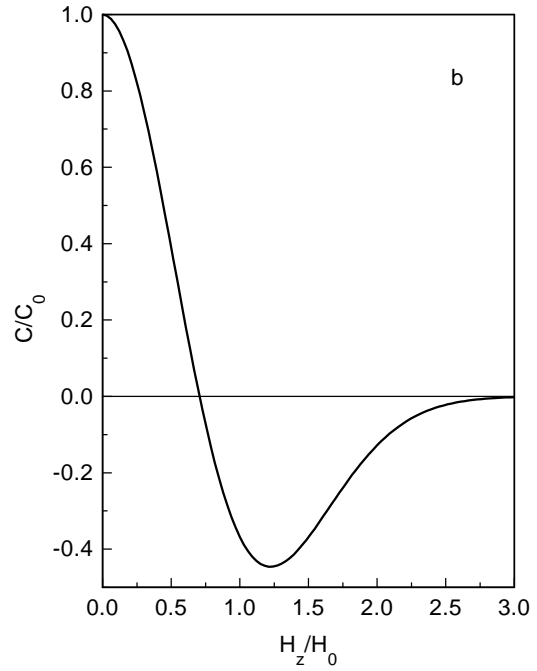
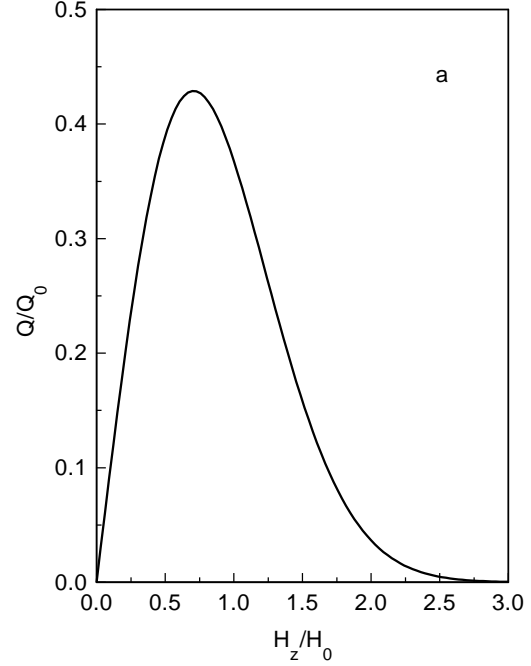


FIG. 1. The behavior of the induced effective charge Q/Q_0 (a) and the flux capacitance C/C_0 (b) in applied magnetic field H_z/H_0 .

At the same time, in view of Eq.(6) the induced polarization is related to the corresponding change of the effective charge $Q \equiv (1/d) \sum_i < q_i x_i >$ in applied magnetic field as follows

$$Q(H_z) = \frac{P_x(H_z)}{d} = Q_0 G(H_z/H_0), \quad (14)$$

where $Q_0 = e\tau J/\hbar$.

It is of interest also to consider the related field behavior of the effective flux capacitance $C \equiv \tau dQ(H_z)/d\Phi$ which in view of Eq.(14) reads

$$C(H_z) = C_0 \left(1 - 2 \frac{H_z^2}{H_0^2} \right) e^{-H_z^2/H_0^2}, \quad (15)$$

where $\Phi = SH_z$, and $C_0 = \tau Q_0/\Phi_0$. Figure 1 shows the behavior of the induced effective charge Q/Q_0 (a) and the corresponding capacitance C/C_0 (b) in applied magnetic field H_z/H_0 . As is seen, at $H_z/H_0 \approx 0.8$ the effective charge reaches its maximum while the capacitance changes its sign at this field, suggesting a significant redistribution of the junction charge balance in a model system under the influence of an applied magnetic field, near the Josephson critical field H_0 . Note that a somewhat similar behavior of the magnetic field induced charge (and related capacitance) has been observed in 2D electron systems [15].

Taking $\tau = 10^{-10}s$ for the Josephson relaxation time (which is related to the Josephson plasma frequency, $\omega_p \simeq \tau^{-1}$, known to be the characteristic frequency of the system for $E_c \ll J$ regime, with E_c being the Coulomb grain's charge energy; typically [7] $\omega_p = 10^9 - 10^{11} Hz$) and $J/k_B = 90K$ for a zero-temperature Josephson energy in *YBCO* ceramics, we arrive at the following estimates of the effective charge $Q_0 \approx 10^{-16}C$, flux capacitance $C_0 \approx 10^{-11}F$, the equivalent current $I_0 \sim Q_0/\tau \approx 10^{-6}A$, and voltage $V_0 \sim Q_0/C_0 \approx 10^{-5}V$. We note that the above set of estimates fall into the range of parameters used in typical experiments to study the charging effects both in single JJs (with the working frequency from RF range of $\omega \simeq 10GHz$ used to stimulate the system [9]) and JJAs [10] suggesting thus quite an optimistic possibility to observe the above-predicted field induced effects experimentally, using a specially prepared system of arrays of superconducting grains.

In summary, a zero-temperature behavior of the induced electric polarization, effective charge and concomitant flux capacitance in an applied magnetic field have been considered within a 3D model of Josephson junction arrays. A possibility of the experimental observation of the discussed effects in artificially prepared system of superconducting junctions was suggested.

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